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MODELING TRANSPORTATION AND STORAGE SYSTEMS IN DEVELOPING AREAS AS CAPACITATED NETWORKS

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Abstract

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Modeling Transportation and Storage Systems in Developing Areas as Capacitated Networks

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Capacitated network models are presented as more appropriate instruments for studying commodity transportation-storage systems in developing areas than traditional linear programming models. Illustrations incorporate realistic features of capacity constraints, multiple transshipment points, storage, intermodal transfer costs and convex costs. A solution is obtained using the efficient Fulkerson Algorithm.

MODELING TRANSPORTATION AND STORAGE SYSTEMS
IN DEVELOPING AREAS AS CAPACITATED NETWORKS*

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Introduction

Many less developed countries (LDC's) are investing vast sums to eliminate bottlenecks and to reduce freight and spoilage costs in agricultural transportation and storage systems. A major problem has been the lack of an appropriate methodology to locate potential bottlenecks in a given transport-storage system and to evaluate the economic impact of selected alternative improvements on overall efficiency.

The purpose of this paper is to present such a methodology and illustrate how it may be used to solve a variety of transportation problems. It briefly reviews three linear programming models commonly used to study commodity transfer problems in developed countries (DC's), pointing out some of their limitations when dealing with problems of major importance in LDC's. The remainder of the paper discusses the capacitated network model as a flexible and useful instrument in studying transportation and storage problems, especially in developing areas. A capacitated network example is formulated and solved (using the Fulkerson algorithm) as an illustration.

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A Brief Review of Three Transportation Models

Agricultural economists have traditionally used three types of models to study the allocation of commodities from surplus (producing) regions to deficit (consuming) regions: 1) the simple transportation model; 2) the transshipment model; and (3) the spatial price equilibrium model.

The simple transportation model is typically restricted to finding least cost solutions in problems such as those involving the shipment of a commodity directly from a series of origins (e.g., factories) to a series of destinations (e.g., warehouses). The transshipment model allows one transshipment point between each origin-destination (O-D) pair. This additional flexibility permits analysis of more complex problems, such as the determination of the optimum combination of processing, storage and interregional commodity movement patterns [King and Henry; Kriebel]. The transshipment model can also be used in optimal location analysis [King and Logan; Rhody; B. Wright: Goldman: Casetti: Ladd and Lifferth].

The spatial price equilibrium model is the only model to represent a theoretical equilibrium of demand and supply, and is thus useful in projecting trade flows where statistics do not permit direct mapping of interregional patterns of trade [Morrill and Garrison; King; Takayama and Judge; Walker].[1] This model has been formulated by Takayama and Judge as a quadratic program to improve its computational efficiency.

These three instruments are normally not used to model the physical characteristics of transport-storage systems. They are limited in this respect

[1]Making price determination endogenous (the advantage of this model) sacrifices the simplicity and relative efficiency of the transportation and transshipment algorithms [King and Logan, p. 96]. Some other modifications discussed in this paper may be made by writing transportation problems as general linear programs. The complexity and inefficiency of such formulations, however, make the capacitated network approach clearly preferable.

by the assumptions that any O-D linkages have infinite capacities, and that no more than one transshipment point may exist between an O-D pair (the transshipment model). To incorporate either maximum capacities on given linkages or multiple transshipment points would imply an exponential increase in the conceptual and computational complexity of the problem. In fact, the incorporation of these real-world features can exhaust computer capabilities on very small problems [Ford and Fulkerson, p. 93].[2] These may not be important limitations for a variety of agricultural applications in DC's, since carriers are seldom saturated by commodities for any length of time, while the diversity and complexity of the systems would in any case restrict modelling to a very small system. There are, however, cases where these limitations are crucial and researchers have turned to the capacitated network approach to resolve them. The applications found in the literature include studies of urban traffic (Gauthier; Muraco), coal shipment in the Great Lakes area (King *et al*.), fruit distribution in New Zealand (Sinclair and Kissling) and containerized shipping on the South Island (McCurdy *et al*.).

The representation of cost and capacity characteristics make the capacitated network approach particularly useful when studying rapidly developing agricultural regions, since it permits the researcher to treat issues such as:

1) the efficiency of the entire transport-storage network;
2) the identification of existing bottlenecks and those which may appear with projected increases in agricultural output;
3) the costs and capacity characteristics of individual links in the network;

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[2]Dantzig was one of the first to recognize the advantages of the capacitated network approach, incorporating it in his 1963 linear programming text.

4) the quantitative effects of specific improvements in the network in terms of accessibility of nodes (centers) within the network and reduction in total shipping costs; and

5) the effect of nonlinear cost functions.

The Capacitated Network Model

The power and simplicity of the capacitated network approach in analyzing a transport-storage system can be best appreciated by considering some illustrations. Figure 1 is a simplified representation of the transportation system in Northwestern São Paulo State and the State of Paraná, Brazil.[3] Londrina (2) is the center of an established producing region and Cascavel (1) is a rapidly expanding frontier area. Both regions are experiencing dramatic agricultural development. The highway system has modern main arteries and is fairly complete, but the rail system is antiquated and serves only part of the area. Grains produced at (1) and (2) meet some domestic demands (estimated exogeneously) at the state capitals São Paulo (5) and Curitiba (6). The remainder is exported through the ports of Santos (7) and Paranagua (8). Cities (3) and (4) will be considered here only as transshipment points.

Figure 2 shows the transportation aspects of Figure 1 as a capacitated network composed of nodes and arcs. A node, i, may represent an origin of flow (producing regions 1 and 2), a transshipment point (3 and 4), or terminal facilities (7 and 8).[4] An arc (i, j) is a linkage between two nodes i and j with shipments permitted from i to j as indicated by arrows.[5] Each arc is

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[3] The examples cited reflect some of the actual transport-storage problems of the area, but are used here only for purposes of illustration. All figures cited are hypothetical.

[4] A node may simultaneously represent a terminal and a transshipment point (5 and 6) or an origin and a transshipment point (2).

[5] Notation in network analysis is not uniform. Conventions adopted in this paper are similar to those used in Potts and Oliver, Taaffe and Gauthier, and King et al.

Figure 1. Simplified Transportation System for Northwestern São Paulo State and State of Paraná, Brazil

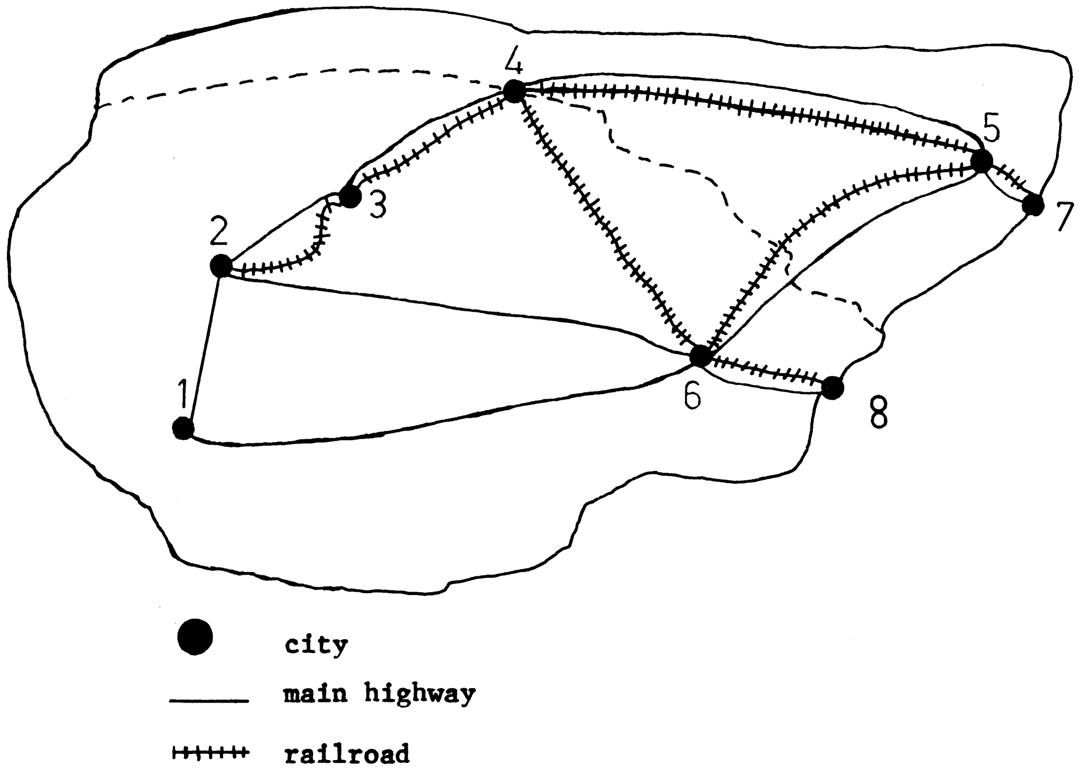
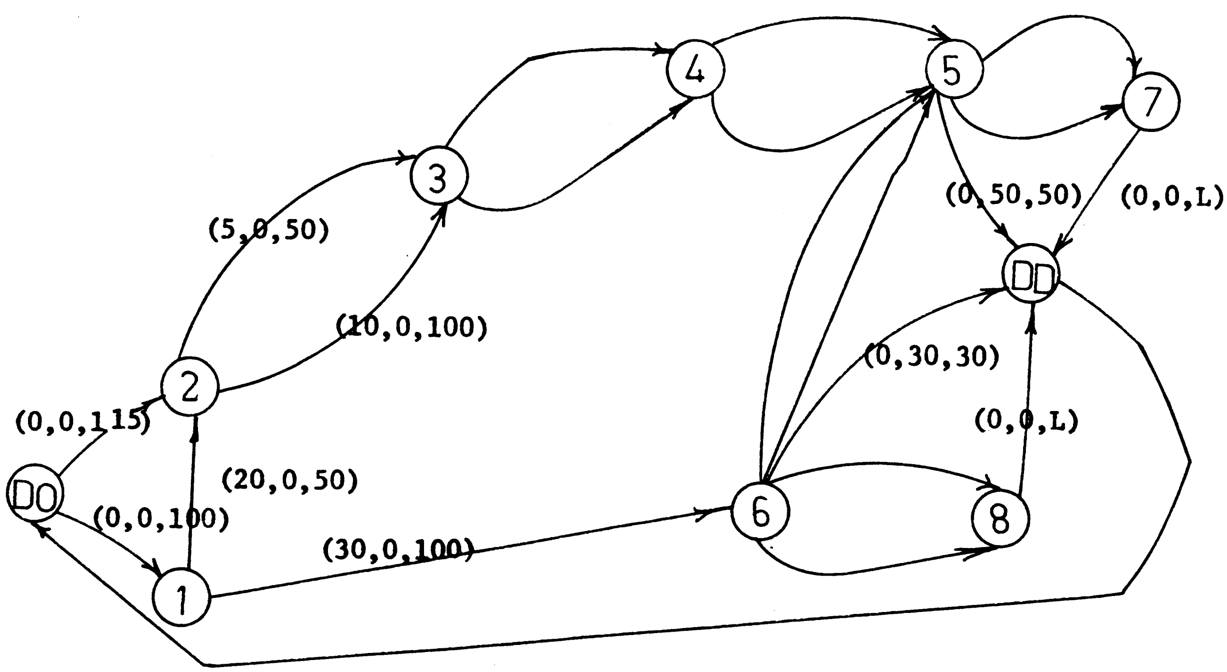


Figure 2. The Transportation System of Figure 1 Depicted as a Capacitated Network



described by its endpoints i and j, and by three parameters (in order): the cost, c_{ij}, of sending a unit of flow between nodes i and j and a lower (l_{ij}) and upper (u_{ij}) bound on the units of flow permitted between i and j during some specified time interval such as a day, month, or year. Capacities are here defined in 10 ton units, and costs in dollars per 10 tons. Any number of arcs may connect the same two nodes as long as the parameters for any two arcs are not all identical.

Several arcs in Figure 2 have been assigned cost and capacity parameters by way of illustration.[6] Node DO is a "dummy" origin which serves as the source of flow for the network. The parameters of the dummy arcs (DO, 1) and (DO, 2) connecting the dummy origin with the "real" origins 1 and 2 indicate that 100 units are available for shipment from producing region 1 and 115 from producing region 2. The zero costs indicate that production costs do not enter into the solution. All production from region 1 can be transported by road to node 6 at $30 per unit, or up to 50 units may be shipped from node 1 to node 2 by road at $20 per unit. From node 2, a maximum of 50 units may move by rail to node 3 at $5 per unit, and an additional 100 units may move by truck for $10 per unit. Node DD is a dummy destination serving as the "sink" for all flows in the network. The lower bounds on the arcs leading to DD are the "demands" (determined exogeneously). The values of 50 on arc (5, DD) and 30 on (6, DD) indicate that 50 units must be sent to node 5 (São Paulo) and 30 units to node 6 (Curitiba). Since the upper bounds are set at the same values, no additional units may flow to these two nodes. Any remaining units which flow through the system will be exported from either of the two ports, as given by the arbitrarily large ("L") upper bounds on arcs (7, DD) and (8, DD). Costs on all dummy arcs are set at zero so they do not influence the optimal solution of the real

[6] All arcs are assigned the three parameters in any real problem. Due to space limitations, problems and procedures for estimating costs and capacities are not discussed.

network. The arc (DD, DO) is explained below.

Intermodal Transfer Costs

The network of Figure 2 makes no allowance for transfer costs between carriers. This assumption is easily relaxed as shown in the subnetwork of Figure 3. Each node is "split" into two nodes, connected by dummy arcs such as (2R, 2) and (2, 2R). The parameters on these arcs indicate that it costs $1 per unit to transfer cargo from truck to rail and $2 from rail to truck. A highway-rail transfer capacity of 30 units exists at location 2, while 50 units can be transferred over all other arcs.

CONCAVE
[redacted] Costs

The assumption of constant unit costs underlies the three traditional transportation models discussed earlier. This is often an unrealistic assumption, since published rail tariffs, for example, are often maximum charges for small shipments. Larger consignments may receive special rates, and unit trains usually receive the lowest rates available.

The capacitated network approach permits relaxation of this assumption as shown in Figure 4 for the rail line linking nodes 2 and 3. Costs are $5 per unit for 9 units or less; $4 per unit from 10-29 units; and $3 per unit from 30 to 50 units. The dummy arc (3, 3') establishes a maximum capacity of 50 units from node 2 to 3 regardless of the size of individual shipments.

Changes in Arcs, Costs and Capacities

The capacitated network model can be easily modified to assess the impact of a) expected increased demand for transportation and storage and b) changing costs and capacities of certain arcs in the system. Such changes are represented simply by changing the respective arc parameters. Changes in relative shipping prices, such as those caused by highway subsidization relative to railways or increases in petroleum prices, are represented in the same fashion. Finally, new facilities are represented by additional arcs. Likewise, the disappearance of facilities

Figure 3. Intermodal Transfer Costs

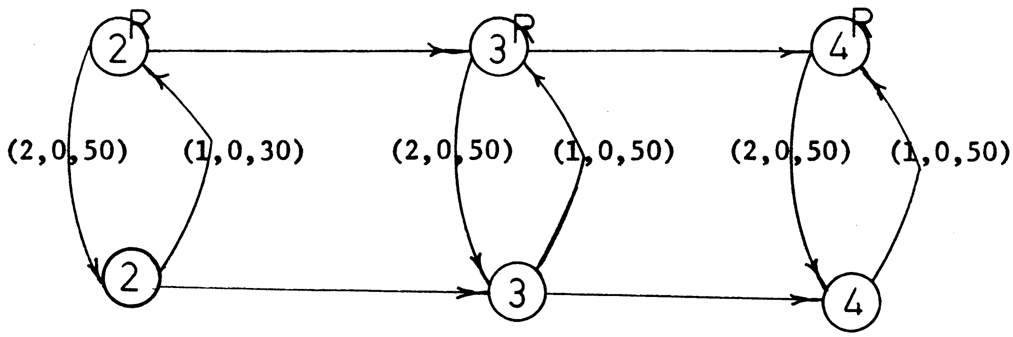


Figure 4. CONCAVE **Costs**

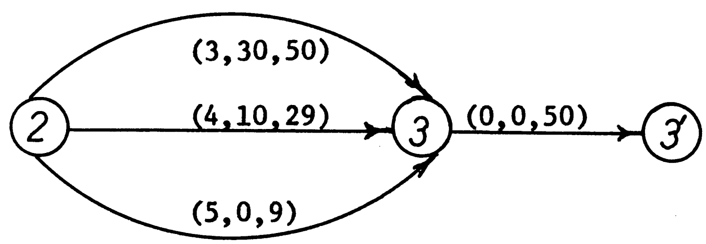
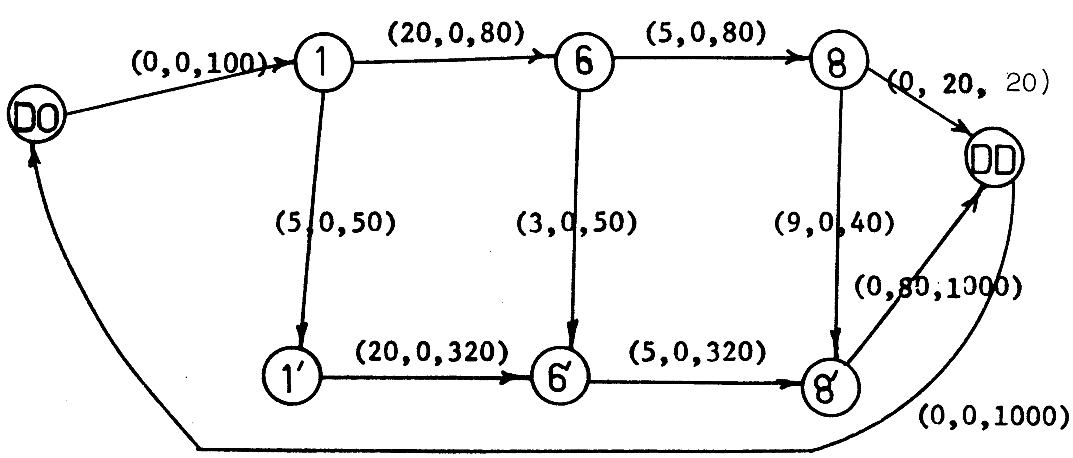


Figure 5. Storage and Storage Costs



such as in rail abandonment is represented by the deletion of affected arcs.

Storage and Storage Costs

The preceding networks were defined for a single time period. Storage, however, can also be represented as a capacitated network either as a separate system or as a complement to transportation. An illustration of a combined transport-storage network is given in Figure 5. Only one aspect of storage is considered: that involving differential transfer costs.[7] Such differentials arise when it is necessary to store a commodity to use low cost carriers that become saturated during the harvest season. They may also arise if storage costs vary among locations (say in ports, due to lack of space or excessive humidity).

In the transport-storage subnetwork of Figure 5, 100 units of (say) soybeans are produced in region 1, but only 20 units are demanded at node 8 during the harvest period. The remaining 80 units must be sent to node 8 during the remainder of the year. Storage is represented by movement of flow over arcs (1,1'), (6,6'), (8,8'). A flow from 6 to 6', for example, indicates storage at node 6 for a specified time period. The flows over arcs (1',6') and (6',8') are actually over the same physical facilities represented by arcs (1,6) and (6,8), but take place during the post-harvest season. The arcs with primed values have greater capacities since the post-harvest season is much longer than the harvest season, giving the transportation facilities more time to move the commodities.

Storage is permitted in the producing region (node 1) at $5 per unit up to 50 units for the post-harvest period, at node 6 ($3 per unit to 50 units), or at the port (node 8) for $9 per unit to 40 units.[8] This example is solved below.

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[7] Storage also occurs due to expected seasonal price increases which are invariant with respect to storage location. The model could be modified to determine the optimum length of storage if the "costs" of price changes were estimated exogeneously and assigned to the storage arcs.

[8] The dummy arcs (8', DD) and (DD,DO) could have been assigned upper capacities of 80 and 100 respectively, without changing the solution. An arbitrarily large upper limit (L = 1,000), however, would not restrict the solution if supply were greater than 100 units and more than 80 units could be shipped to node 8' after the harvest. This frequently occurs in multiple origin-multiple destination models.

An Efficient Solution to the Capacitated Network Problem

The Fulkerson "Out-of-Kilter" Algorithm (OKA) is an efficient instrument for solving capacitated network problems even for very large networks [Potts and Oliver; Durbin and Kroenke; Ford and Fulkerson].[9] All parameters must be established exogeneously. Supply may be equal to or greater than the sum of the amounts demanded. The algorithm determines the maximum set of flows, x_{ij}, so as to minimize the total transfer costs including transport, storage and other costs assigned to the arcs.[10]

The OKA determines the flows, x_{ij}, which minimize total transfer costs (transport, storage and other costs to the arcs). Formally, the OKA minimizes

$$\sum_{ij} c_{ij} \, x_{ij} \quad \text{for all i and j}$$

subject to:

$$l_{ij} \leq x_{ij} \leq u_{ij} \quad \text{for all i and j}$$

and

$$\sum_{j} x_{ji} - \sum_{j} x_{ij} = 0 \text{ for all i}$$

where all symbols are defined as previously.

This last condition is the conservation of flow principle that the total flow into a node must equal the total flow out of it. Thus, in order to solve the problem of Figure 5, a dummy arc (DD, DO) must be added to complete the

[9] A program generously made available to the authors by Dr. H.L. Gauthier of The Ohio State University is designed to handle up to 1,000 nodes and 3,000 arcs. Modern computer capacities permit expansion of capacity beyond this if necessary, and additional efficiencies have been suggested by Wollmer.

[10] The maximum flow is determined by the minimal cut-set [Potts and Oliver, p. 43]. If all "supply" can be forced through the network, the supply arcs constitute the cut-set (i.e., the maximal flow = available supply). Thus the maximal flow is a given flow and will be allocated to the least cost arcs.

system, avoiding loss of flow at the source (DO) and gain of flow at the sink (DD).

The optimal solution to the problem of Figure 5 was determined by the OKA and is given in Table 1. The x_{ij} values are the flows over the arcs (i, j) which constitute the least cost means of forcing the given flow through the network (e.g., 30 units are sent from node 1 to 6 in the post-harvest period as given by $X_{1'6'} = 30$ on arc (1', 6'). Besides the x_{ij}'s, node prices, net arc costs and kilter numbers are determined endogenously.

Node prices, π_i, are recalcuated at each iteration so that increases in commodity flow are along the least expensive paths. They are relative prices and indicative of locational advantages or rents. For example, the price at node 6 is \$20 more than at node 1, reflecting its more favorable location with respect to the destination (node 8).

Table 1. Optimal OKA Solution For Transport-Storage Problem of Figure 5 [a]

Arcs i	j	Cost Per Unit	Lower Limit (Units)	Upper Limit (Units)	x_{ij} Optimal Flows	Net Arc Cost (CBAR)	Kilter Number	Total Transport Costs on Arc (C_{ij} times X_{ij})
DO	1	0	0	100	100	0	0	0
1	1'	5	0	50	30	0	0	150
1	6	20	0	80	70	0	0	1400
1'	6'	20	0	320	30	0	0	600
6	6'	3	0	50	50	-2	0	150
6	8	5	0	80	20	0	0	100
6'	8'	5	0	320	80	0	0	400
8	8'	9	0	40	0	4	0	0
8	DD	0	20	20	20	25	0	0
8'	DD	0	80	1000	80	30	0	0
DD	DO	0	0	1000	100	0	0	0

Total Transfer Cost = \$2,800

[a] Node prices (π_i) are \$0 for nodes Do, 1, and DD; \$5 for node 1'; \$20 for 6; \$25 for 6' and 8 and \$30 for 8'.

The net arc cost (CBAR) is defined as:

(1) $\bar{c}_{ij} = (\pi_i - \pi_j) + c_{ij}$

Negative CBAR values imply that the flow over the arc is at its maximum value and that savings could be obtained if the capacity was expanded and flows diverted from more costly paths. Thus, the arcs with the largest negative CBAR values constitute major bottlenecks to a more efficient transfer of goods and are useful instruments for post-optimal (sensitivity) analysis. The only bottleneck in the system of Figure 5 is arc (6, 6'), that is, storage at node 6. If that capacity were increased, flow could be rerouted so as to meet the demands at a cost reduction of $2 per unit until a bottleneck developed on another arc. Positive CBAR values imply flow is at the minimum value. These values represent the cost to the system of increasing flow over the associated arcs by one unit. If the arcs indicate demand requirements, this is the total cost of sending one additional unit of flow through the system. For arc (8,DD), this cost is $25.

The last endogenous variable is the kilter number of the arcs. All kilter numbers are zero in the optimal solution. A positive kilter number indicates an arc has a non-optimal or infeasible flow, and at least one arc is "out-of-kilter" (i.e., has a positive kilter number) until the optimal solution is obtained (hence the name of the algorithm).

The efficiency of the OKA arises from its rapid convergence to the optimal solution and the ease of post-optimal analysis. The optimal solution of a given problem furnishes a starting point for a subsequent problem when some arc parameters have been altered or arcs added or deleted. The algorithm also provides a highly efficient solution to the simple transportation, one intermediate point transshipment, the shortest path and maximal flow problems as special cases.

Summary and Conclusions

Transportation models developed to date have been most effectively used in studying problems of developed economies. Transport-storage networks in LDC's, however, frequently require the analysis of multiple transshipment points and capacity constraints which are subject to rapid alteration due to massive investment programs. The capacitated network approach outlined in this paper offers the possibility of effectively dealing with these complex problems. The efficiency and flexibility of the OKA solution suggests it may also be preferable to other algorithms like linear programming for the simple transportation problem as well as the transshipment model with a single transshipment point between O-D pairs.

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